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Ghost resonance in a semiconductor laser with optical feedback

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Abstract. – We show both experimentally and numerically a *ghost* resonance in the sudden power dropouts exhibited by a semiconductor laser subject to optical feedback driven by two simultaneous weak periodic signals. The small signal modulation conspires with the complex internal dynamics of the system to produce a resonance at a *ghost* frequency, *i.e.* a frequency that is not present in the driving signals. This is an eminently nonlinear effect not reported before and agrees with the recent theoretical predictions by Chialvo *et al.* (*Phys. Rev. E*, **65** (2002) 050902(R)).

The response of dynamical systems to external driving is a far-reaching problem, with implications ranging from signal detection by sensory systems [1] to information encoding through diode laser modulation in communication systems [2]. In the former context, for instance, recent research efforts have been addressed to understand the perception of complex sounds in auditory systems. To that end, the response of excitable threshold devices to multi-frequency signals has been theoretically shown to exhibit a resonance at a frequency which is absent in the input driving [3]. The present letter reports an experimental realization of this *ghost resonance* in a different type of complex dynamical system, namely a semiconductor laser subject to optical feedback. This system has attracted much attention of the researchers for more than three decades. One of its most interesting characteristic regimes is the low-frequency fluctuation regime (LFF), in which the output power of the laser suffers sudden dropouts to almost zero power at irregular time intervals when biased close to threshold [4]. Although the LFF behavior was already observed at the end of the seventies, its dynamics is not fully understood yet.

Recent experimental [5] and numerical [6, 7] reports show the conditions for which a laser subject to optical feedback and biased close to threshold is able to operate in an excitable

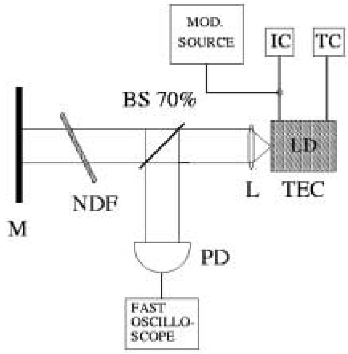

Fig. 1 – Diagram of the experimental setup.

regime, before the onset of the LFFs. This means that a laser prepared in such a state is stable under small periodic perturbation of the bias current and exhibits the three ingredients of any excitable system, namely: the existence of a threshold for the perturbation amplitude above which the dropout event can occur; the form and size of the dropout events are invariant to changes in the magnitude of the perturbation, although multipulse emission has also been predicted [8]; and a refractory time exists: if a second perturbation is applied at a time shorter than the refractory time, the system no longer responds.

It has also been shown both experimentally [9, 10] and numerically [11, 12] that a laser subject to optical feedback can also exhibit stochastic [13] and coherence [14] resonance when biased close to threshold, extending the richness of the dynamical behaviors of this system. Stochastic resonance is characterized by an optimum coherence of the system output with a weak periodic signal for an intermediate value of the noise level. On the other hand, coherence resonance is characterized by an almost periodic response of the system to an intermediate level of noise but without any external periodic signal. Both effects have also been observed in a large variety of systems including periodic and chaotic systems [13, 15].

Recently, it has also been shown that the laser responses can be entrained to give a periodic train of dropouts by superimposing an external forcing to a bias current close to threshold. If the amplitude of the forcing is larger than a certain value, the dropouts occur at the frequency of the external forcing when the latter has a frequency larger than the mean frequency of the dropouts in the absence of the perturbation [16–18].

In all the previous studies, semiconductor lasers were excited at most with a *single* sinusoidal input. In this letter we go further and study experimentally and numerically the response of a semiconductor laser subject to optical feedback biased close to threshold modulated by two weak sinusoidal signals.

Analyzing this kind of driving can be considered a first step towards an understanding of the influence of complex signals on this system. Two-frequency forcing of dynamical systems has long been studied [19] with an emphasis being usually placed on quasiperiodic dynamics. In contrast, our results show a resonance at a frequency that is absent in the input signals, which we thus call *ghost* resonance. We describe the conditions for and the location of this *ghost* resonant frequency, which has recently been predicted, for a simpler system, by means of theoretical arguments in ref. [3].

The experimental setup, shown in fig. 1, consists of an index-guided AlGaInP semiconductor laser (Roithner RLT6505G), with a nominal wavelength of 658 nm. The threshold current is $I_{\rm th} = 18.4\,{\rm mA}$ for a temperature of $19.86 \pm 0.01\,^{\circ}{\rm C}$. The injection current (IC), without

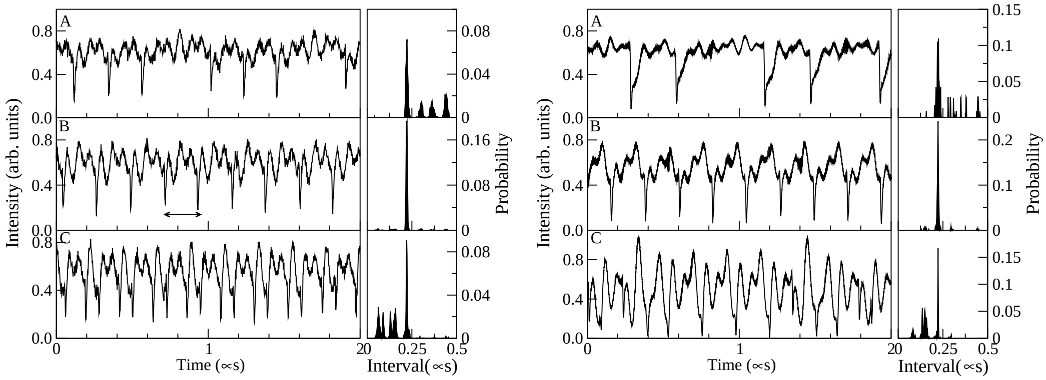

Fig. 2 – Left panels: experimental, right panels: numerical results. Time series of the optical power in response to low (A), medium (B) and high (C) amplitudes of the injected signals. The PDFs of the dropouts intervals at the three amplitudes are also shown. The PDFs largest peak corresponds to $1/f_0$. In all cases the driving signal contains two frequencies (see text).

modulation, is set to 19.7 ± 0.1 mA all through the experiment. An antireflection-coated laser-diode objective (L) is used to collimate the emitted light. An external mirror (M) is placed 83.5 cm away from the front facet of the laser, introducing a delay time of $\tau \sim 5.56$ ns. The feedback strength is such that it yields a threshold reduction of 7.0% and it is adjusted by placing a neutral density filter (NDF) in the external cavity. The output intensity is collected by a fast photodetector (PD) and analyzed with a 500 MHz bandwidth acquisition card.

We are interested in the system response to modulation composed of multiple periodic signals $f_1, f_2, \ldots, f_n$. Although the present letter focus mostly in the case of two components, the driving signal has the following general form:

$$\begin{aligned} I(t) \; &= \; I_{\mathrm{b}}\big\{1 + m\big[\sin(2\pi(kf_0t + \Delta ft)) + \sin(2\pi((k+1)f_0t + \Delta ft)) + \\ &\qquad + \cdots + \sin(2\pi((k+n-1)f_0t + \Delta ft))\big]\big\}, \end{aligned} \tag{1}$$

with $k > 1$ and n being the number of terms used. I_{b} is the bias current and m is the modulation amplitude. Here we choose to use two terms ($n = 2$) and $f_0 = 4.5$ MHz (although the same qualitative features would be observed for other choices of f_0). For simplicity, initially we describe results for $\Delta f = 0$, *i.e.*, the singular case of harmonic signals.

The operating parameters of the system are chosen in such a way that in the absence of modulation the laser emits a continuous-wave (CW) light intensity. Power dropouts start to appear when a small amount of modulation is added to the laser pump current [20]. Figure 2 shows representative time traces and probability distribution functions (PDF) of dropout events. The left plot of the figure corresponds to experimental data for low ($m = 0.057$), intermediate ($m = 0.0815$) and high ($m = 0.114$) amplitude values of the injected signals. It can be clearly seen that for the intermediate amplitude the dropouts are almost equally spaced at a time interval that corresponds grossly to $1/f_0$ (depicted by the double-headed arrow in the middle panel), a frequency that is not being injected. Thus the laser is detecting the subharmonic frequency in a nonlinear way. To better visualize this fact, we plot the PDFs for a large number of dropouts (approximately 1500). For the small amplitude (top-right panel in each side) one can observe a peak at a time $1/f_0$ and other peaks at longer times which indicate that the system responds sometimes to f_0 although at some others times dropouts are skipped. For the optimum value of the amplitude (middle-right panel in each side) the PDF

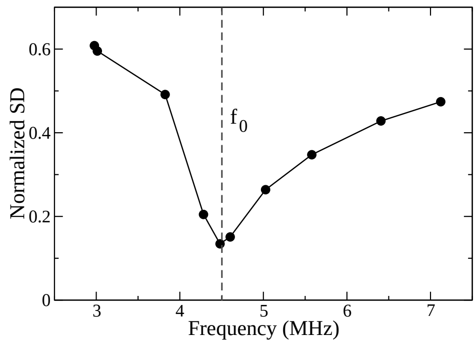

Fig. 3 – Experimental results showing that the variability of the dropout intervals reaches a minimum when its frequency approaches f_0.

has a clear peak at $1/f_0$ which indicates that the system is resonating with this frequency. For the higher amplitude (bottom-right panel in each side), there are several peaks at different times corresponding to higher frequencies.

The resonance with the ghost frequency can be visualized by measuring the mean interval between dropout events and its standard deviation (SD) at various values of the signal amplitude m. Figure 3 shows these results plotted as the normalized SD (*i.e.* SD/mean) as a function of the mean frequency of dropout events. It is clearly seen that the minimum coincides with the f_0 (vertical dashed line), *i.e.*, the ghost frequency.

The ghost frequency is not, as one naively would expect, simply the difference between the two components f_1 and f_2 (where $f_1 = 2f_0$ and $f_2 = 3f_0$). This is demonstrated by adding a small term $\Delta f \neq 0$ which shifts both frequencies equally, makes them incommensurate, and renders the signal inharmonic [3]. In this case we observe that the resonant frequency shifts as well, despite the fact that the difference remains constant. Results from experimental runs using $f_1 = 7$ to $10.5\,\mathrm{MHz}$ and $f_2 = 11.5$ to $15\,\mathrm{MHz}$ and selecting the optimum amplitude $m = 0.0815$ are presented on the left side of fig. 4. The format of the plot is meant to illustrate

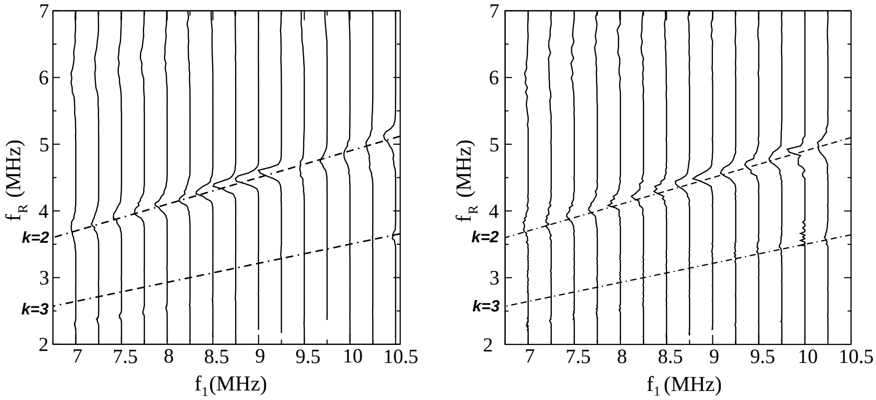

Fig. 4 – Right side: experimental, left side: numerical results. PDFs of the intervals between dropouts are plotted as their inverse. For each pair of driving f_1-f_2 frequencies explored the resulting PDF is plotted at the corresponding f_1 frequency. The lines are the expected resonance frequencies from the theoretical prediction given in the text.

better the linear change of the resonant frequency f_R as a function of the frequency shift. The PDFs are plotted using the frequency (*i.e.*, inverse of the dropout intervals) axis and they are lined up with the f_1 frequency at which they were obtained. It can be seen that the density of the most frequent dropouts lies on a straight line. The experimental results show a remarkable agreement with the prediction given in [3], given by $f_R = f_0 + \Delta f/(k + 1/2)$. Since the range of f_1 we explored is about twice f_0, the dotted line labeled "$k = 2$" predicts the location of the most important resonance and the one labeled "$k = 3$" the expected ones if the range were to be extended further up. Thus, the results presented in this figure agree extremely well with the ones described previously in a simpler system in [3] and it is the first experimental demonstration of this type of resonance at the ghost frequency.

We have also checked that our experimental results can be reproduced by the well-known Lang-Kobayashi (L-K) model [21], which is the simplest model to describe the dynamics of a semiconductor laser subject to weak/moderate optical feedback. The L-K equations account for single-mode operation and describe the time evolution of the slowly varying amplitude of the electric field $E(t)$ and the excess carrier number $N(t)$:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{1+i\alpha}{2}(G(E,N)-\gamma)E(t) + \kappa e^{-i\omega\tau}E(t-\tau) + \sqrt{2\beta N}\xi(t), \tag{2}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = I_\mathrm{b}\big(1 + m\big\{\sin(2\pi(kf_0t + \Delta ft)) + \sin(2\pi((k+1)f_0t + \Delta ft))\big\}\big) - \gamma_e N(t) - -G(E,N)|E(t)|^2. \tag{3}$$

The first term on the right-hand side of eq. (2) accounts for the stimulated emission. $\alpha = 3.4$ is the linewidth enhancement factor and $\gamma = 0.24\,\mathrm{ps}^{-1}$ is the cavity decay rate. The second term is the feedback term which is described by two parameters: the feedback strength $\kappa = 20\,\mathrm{ns}^{-1}$ and the external round-trip time $\tau = 5.57\,\mathrm{ns}$. $\omega/2\pi = 4.56 \times 10^{14}\,\mathrm{Hz}$ is the laser free running frequency. The last term accounts for the spontaneous-emission noise, considered as a Gaussian white-noise source of zero mean and delta correlation, with a spontaneous-emission rate $\beta = 5 \times 10^{-10}\,\mathrm{ps}^{-1}$. The first term in eq. (3) accounts for the injection current with the two sinusoidal inputs at frequencies $2f_0$ and $3f_0$, being $f_0 = 4.5\,\mathrm{MHz}$ and the modulation amplitude $m = 0.0118$ with respect to threshold. The second term accounts for the spontaneous recombination and the third one for the stimulated recombination. $I_\mathrm{b} = 1.26 \times 10^5\,\mathrm{ps}^{-1}$ is the pump parameter, which corresponds to a laser pumped 1.015 times above threshold, with $I_\mathrm{th} = 19.8\,\mathrm{mA}$. The carrier decay rate is $\gamma_e = 0.62\,\mathrm{ns}^{-1}$. The material gain $G(E,N)$ depends linearly on N and is slightly nonlinear on $|E|^2$, according to the expression $G(E,N) = g(N(t)-N_0)/(1+s|E(t)|^2)$, where $N_0 = 1.5\times10^8$ is the number of carriers at transparency, g is the differential gain coefficient, and $s = 1\times10^{-7}$ is the saturation gain coefficient.

On the right side of fig. 2 we plot the time traces and PDFs obtained from the model in the same conditions of the experimental ones. A clear correspondence can be observed. Similarly, the right side of fig. 4 shows the results of the inharmonic case obtained by numerical simulations. It can be clearly seen that the same scaling is obtained as in the experiments, which indicates that the L-K model is also able to extract the main features of this new resonant phenomenon. The power spectra of the time series of the optical power (corresponding to the middle rows in fig. 2) are shown in fig. 5. The power spectrum of the modulated input current shown in panel a) shows the absence of the frequency f_0 which is distinctly present in the output (plots b) and c)).

The theoretical model also helps us to confirm that the experimentally observed behavior is not a simple linear subharmonic resonance. To this end, we analyze numerically the response of the system to a three-frequency inharmonic signal ($n = 3$ in eq. (1)) with $m = 0.008$. The

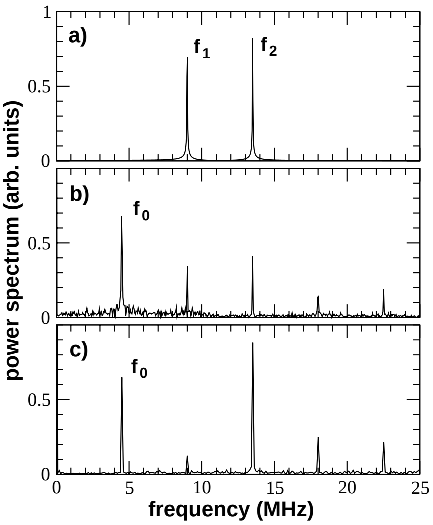

Fig. 5

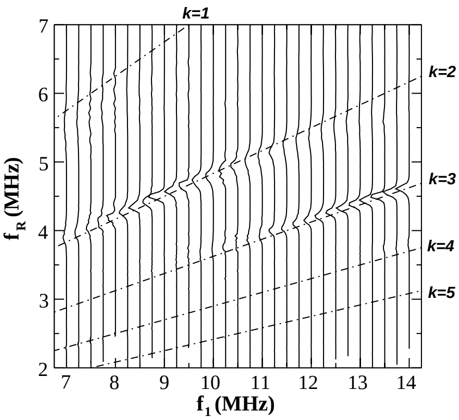

Fig. 6

Fig. 5 – Power spectra of the time series of (a) input pump signal; (b) output optical power obtained from the numerical simulations; and (c) output optical power obtained from the experiment. $f_1 = 9\,\mathrm{MHz}$ and $f_2 = 13.5\,\mathrm{MHz}$. Notice that f_0 only appears in the outputs.

Fig. 6 – Numerically determined probability distribution of the intervals between dropouts for three-frequency forcing plotted using the same format as in fig. 4. Dot-dashed lines represent the theoretical predictions for $k = 1$–5.

results are shown in fig. 6, and compared with the theoretical prediction [3], which for $n = 3$ is $f_\mathrm{R} = f_0 + \Delta f/(k + 3/2)$. The agreement is also quite satisfactory.

Under the current experimental conditions it is cumbersome to change the noise intensity, and thus one is unable to fully explore the stochastic aspects of this resonance, as was done in [3]. We find that the most robust results are obtained when the bias is tuned close to the threshold for LFF, a region where the effects of even minute fluctuations are expected to be magnified. The origin of these fluctuations, whether they are induced by the internal nonlinear dynamics or by stochastic sources, remains unclear. The consequences of these aspects deserve to be explored in future work. The bases of the ghost resonance were discussed previously in ref. [3], where it was argued that the simple linear interference of the two (or three, four, ...) sinusoidal inputs generate peaks with larger amplitude at time intervals close to $1/f_0$ (for the case of harmonics signals), which are detected nonlinearly by means of a threshold. In the present case, the laser's intrinsic nonlinearities are playing the role of the threshold of the simple model analyzed in [3]. In this sense, this phenomenon is shown to be rather ubiquitous and it can thus be expected to arise in other nonlinear systems with excitable properties.

In conclusion, we have described, experimentally and numerically, a new type of resonance observed when a semiconductor laser subject to optical feedback is biased close to its excitable dynamics, near the onset of the low-frequency fluctuation regime. It is shown that, when this system is modulated with two weak periodic signals of different frequencies, it exhibits a resonance at a *ghost* frequency, *i.e.*, a frequency that it is not present in the modulating input. We find that for injection frequencies kf_0 and $(k+1)f_0$, f_0 being any slow frequency, we observe the resonance at exactly f_0, a frequency that is not present in the injection current.

It is also observed that, when a constant shift is added to both frequencies of the injected signal, the resonance does not appear at the difference between the two frequencies but at a frequency that follows a simple linear relationship. Similar results are obtained numerically for three-frequency forcing. Our results confirm the recent theoretical predictions by Chialvo and coworkers, based on a simpler system [3].

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