Demultiplexing Chaos From Multimode Semiconductor Lasers

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***Abstract*—We show numerically that the injection of two chaotic modes of a multimode semiconductor laser with optical feedback into two single-mode stand-alone semiconductor lasers leads to chaotic synchronization between the respective intensities. The effect of parameter mismatch between the transmitter and receiver lasers is examined, and it is concluded that the observed synchronization is a consequence of injection locking. Under these conditions, the possibility of using this demultiplexing scheme for message transmission is examined.**

***Index Terms*—Chaos synchronization, chaotic communications, multimode lasers, parameter mismatch, semiconductor lasers.**

I. Introduction

THE USE of multimode lasers for multichannel chaotic communications [1] is a straightforward generalization of the single-mode, single-channel chaotic communication setups that have been profusely studied in the last decade [2]. Several works have focused in the multimode dynamics of chaotic semiconductor lasers [3]–[6]. Nevertheless numerical [7], [8] and experimental studies of multimode chaotic synchronization [9] are so far scarce.

In a recent experimental analysis, Lee and Shore [10] have studied the injection of light from a multimode Fabry–Perot (FP) semiconductor laser into two distributed-feedback (DFB) lasers, whose wavelengths are matched to two different modes of the transmitter. That work differs with previous studies of multimode semiconductor laser synchronization [7], [11], [12] in the sense that only the transmitter is multimode while the receiver is single-mode, and therefore, they are very different dynamical systems from one another. The experimental results of Lee and Shore indicate that a high degree of synchronization between the FP transmitter and the DFB receivers is achieved. This might seem surprising at first glance, since chaotic synchronization usually requires that the two coupled oscillators involved are very similar to each other.

Chaotic communications are based on the synchronization between transmitter and receiver, with the receiver synchronizing only with the chaotic carrier and not with the message. Therefore subtracting the receiver output (chaotic carrier) to the receiver input (chaotic carrier + message) it is possible to recover the encrypted message. The results obtained by Lee and Shore open the question of how secure is chaotic synchronization of semiconductor lasers as a technique of message encryption, since they seem to indicate that an eavesdropper with a laser very different to the transmitter would be able to recover the message.

In order to put the experimental results of Lee and Shore [10] in perspective and analyze the feasibility of this scheme as a technique of message recovery, we have undertaken a numerical investigation of a similar version of that experiment, in which a multimode laser model of the Lang–Kobayashi type is used to drive two single-mode laser models representing the DFB receivers of the experiments. Since we are mainly concerned about the possibility of synchronizing two different lasers, we choose a coupling scheme simpler than the one of Lee and Shore, in which the driving signals are now the individual modes matched to the receivers, instead of the total intensity of the multimode transmitter. In spite of the different coupling schemes, the model produces results qualitatively identical to the experimental observations of Lee and Shore [10]. The synchronization is found to be robust to mismatches between the internal parameters of emitter and receivers, something that certainly exists in the experiment. On the other hand, a joint analysis of the cross correlation and synchronization error of the time series indicates that the synchronization is a consequence of nonlinear amplification due to the strong injection. Finally, we have examined the possibility of using the observed synchronization for chaotic communications. The fact that synchronization is achieved even when considering different lasers with different internal parameters may indicate that the message could be recovered by an eavesdropper. On the contrary, our results indicate that although synchronization is achieved, message decoding within the proposed scheme is not effective when encoding is done via the injection current of the multimode laser, which confirms the security of this method of encryption.

II. Model

We consider the configuration shown in Fig. 1. The output intensity of a multimode semiconductor laser with external optical feedback is unidirectionally injected into two different single-mode semiconductor lasers in an open loop configuration (without feedback). In fact, two modes of the transmitter laser are injected into the two receivers, in a way that each injected mode has the same wavelength as its corresponding receiver laser. The output signal of the transmitter is sent to a grating in order to separate the different modes and inject them into the corresponding receiver.

We use a standard multimode model to describe the evolution of the slow-varying complex envelope $E_m^{\rm t}(t)$ of the mth

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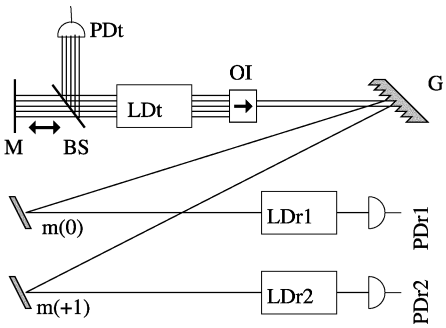

Fig. 1. Schematic setup of the communication system considered. Each horizontal line represents a longitudinal mode emitted by the multimode transmitter laser diode LD-t. The transmitter laser is subject to optical feedback from the external mirror M. An optical isolator OI ensures unidirectional propagation from the transmitter laser modes to the receiver lasers LD-r1 and LD-r2. A grating G is used to allow selective injection of only one longitudinal mode into each receiver, where the wavelength of the injected mode corresponds to that of the receiver. The dynamical behavior of both receiver lasers can be measured by the photodiodes PD-r1 and PD-r2. In the transmitter laser, a beam splitter BS may be used to direct part of the laser output to the photodetector PD-t.

longitudinal mode of the electric field generated by the transmitter and the corresponding carrier number $N^{\rm t}(t)$, assuming that the carriers are shared by all modes. The model is obtained from an extension of the Lang–Kobayashi (L-K) equations describing the dynamical behavior of semiconductor lasers with optical feedback [13]

$$\frac{dE_m^{\rm t}}{dt} = \frac{1}{2}(1+i\alpha^{\rm t})\left[G_m(N^{\rm t})-\gamma^{\rm t}\right]E_m^{\rm t}(t) + \kappa_t E_m^{\rm t}(t-\tau_t)e^{-i\omega_{0m}\tau_t} + F_m^{\rm t}(t) \quad (1)$$

$$\frac{dN^{\rm t}}{dt} = \frac{I^{\rm t}}{q} - \gamma_e^{\rm t}N^{\rm t}(t) - \sum_{m=-M}^{M} G_m(N^{\rm t})\left|E_m^{\rm t}\right|^2 \quad (2)$$

where the transmitter field in (1) is subject to its own delayed feedback.

The dynamics of the two single-mode receiver lasers, named LD-r1 and LD-r2, is described by means of a standard rate equation model with delayed injection and, in principle, with different parameters from those of the transmitter laser

$$\frac{dE^{(1,2)}}{dt} = \frac{1+i\alpha^{(1,2)}}{2}\left[G^{(1,2)}\left(N^{(1,2)}\right)-\gamma^{(1,2)}\right]E^{(1,2)} + \kappa_c e^{\omega_{0m}\tau_c}E^{\rm t}_{[\rm m(0),m(+1)]}(t-\tau_c) + F^{(1,2)}(t) \quad (3)$$

$$\frac{dN^{(1,2)}}{dt} = \frac{I^{(1,2)}}{q} - \gamma_e^{(1,2)}N^{(1,2)}(t) - G^{(1,2)}\left(N^{(1,2)}\right)\left|E^{(1,2)}\right|^2 \quad (4)$$

where the delayed term in the receiver field equations (3) corresponds to the injection from the transmitter modes. We assume zero-detuning between the two receiver lasers and its corresponding injected modes in order to simplify the model. The total number of modes in the transmitter laser is $2M+1$. The electric field amplitudes $E_m(t)$ are normalized so that $P_m(t) = |E_m(t)|^2$ measures the photon number in the mth mode. The intrinsic laser parameters are the linewidth enhancement factor $\alpha^{\rm t,1,2}$ and the mode-dependent cavity loss $\gamma^{\rm t,1,2}$. Spontaneous emission fluctuations are represented by a Langevin noise force

TABLE I
PARAMETER VALUES USED IN THE SIMULATIONS

Description		Value
Linewidth enhancement factor	α	3.5
Cavity decay rate	γ	0.238 ps^{-1}
Carrier decay rate	γ_e	6.21×10^{-4} ps^{-1}
Spontaneous emission noise	β	0.5×10^{-10} ps^{-1}
Injection current	I	$1.100\times I_{th}$
Threshold current	I_{th}	19.81 mA
Internal round-trip time	τ_L	8.5 ps
Material gain width	$\Delta\omega_g$	$2\pi\times2.117$ THz
Saturation coefficient	s	0.0
Differential gain	g_c	3.2×10^{-9} ps^{-1}
Transparency inversion	N_0	1.25×10^{8}
Transmitter feedback level	κ_t	0.025 ps^{-1}
Transmitter feedback time	τ_t	1.0 ns
Coupling level	κ_c	variable
Coupling time	τ_c	0.0 ns

$F(t) = \sqrt{2\beta N}\xi(t)$, being $\xi(t)$ a Gaussian white noise term of zero mean and unity intensity and β measuring the noise strength. In the equations for the carrier densities, $\gamma_e^{\rm t,1,2}$ is the inverse lifetime of the electron–hole pairs, $I^{\rm t,1,2}$ is the injection current.

The feedback parameters of the transmitter, namely the feedback level κ_t and the round-trip time of the external cavity τ_t, are assumed to be equal for all modes. The phase shift $\omega_{0m}\tau_t$ appearing in the feedback term is due to the external-cavity round-trip, with ω_{0m} representing the nominal frequency of the mth mode, i.e., $\omega_{0m} = \omega_c \pm m\Delta\omega_L$, where ω_c is the frequency of the gain peak of the solitary laser (corresponding to the mode $m_c = 0$) and $\Delta\omega_L$ is the longitudinal mode spacing given by $\Delta\omega_L = 2\pi/\tau_L^{\rm t}$, where $\tau_L^{\rm t,1,2}$ is the internal round-trip time. For the sake of simplicity, coupling between the transmitter and receiver lasers is considered instantaneous (i.e., $\tau_c = 0$). The mode-dependent gain coefficient G_m of the transmitter is assumed to have a parabolic profile with its maximum centered at m_c

$$G_m^{\rm t}(N^{\rm t}) = \frac{g_c^{\rm t}\left(N^{\rm t}-N_0^{\rm t}\right)}{1+s^{\rm t}\sum_{-m}^{m}|E_m(t)|^2}\left[1-\left(\frac{m\Delta\omega_L}{\Delta\omega_g}\right)^2\right] \quad (5)$$

where $g_c^{\rm t}$ is the differential gain coefficient at the peak gain of the solitary laser $m = 0$, $\Delta\omega_g$ is the gain width of the laser material, N_0 is the carrier number at transparency, and $s^{\rm t}$ is the saturation term. The gain term for the receiver lasers is also a nonlinear function given by

$$G^{(1,2)}\left(N^{(1,2)}\right) = \frac{g_c^{(1,2)}\left(N^{(1,2)}-N_0^{(1,2)}\right)}{1+s^{(1,2)}\left|E^{(1,2)}(t)\right|^2} \quad (6)$$

where the parameters $g_c^{(1,2)}$ and $s^{(1,2)}$ have the same meaning as in (5). For simplicity, in what follows we ignore saturation effects for both the transmitter and the receivers (i.e., $s^{\rm t,1,2} = 0$).

In the calculations presented in what follows, we assume a transmitter laser with five active optical modes (i.e., $M = 2$). The laser parameters used in the simulations are those indicated in Table I, with the laser parameters assumed identical between the transmitter and receivers models in the next Section. The more realistic case of parameter mismatch will be considered in Section IV.

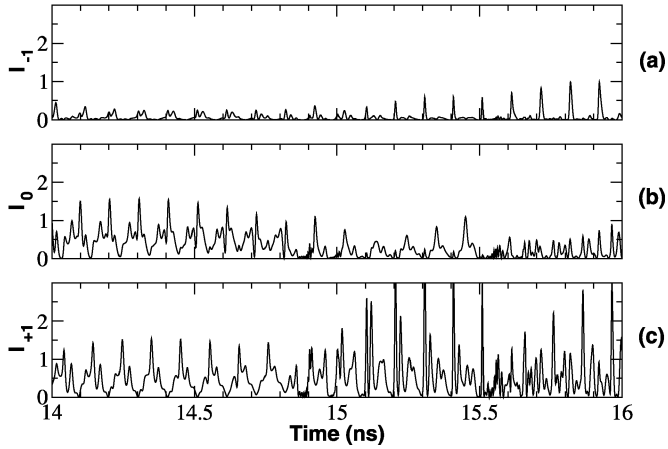

Fig. 2. Output intensity of the three dominant modes of the transmitter laser at fast time scale: (a) $m = -1$, (b) $m = -0$, (c) $m = +1$. We can observe the out-of-phase dynamics of the longitudinal modes.

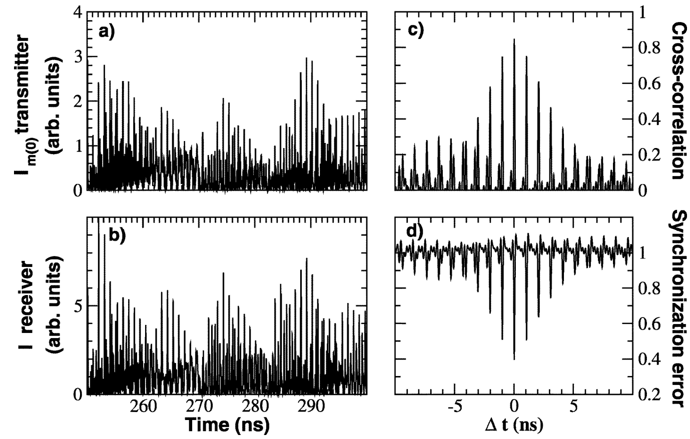

Fig. 3. Output intensity of (a) transmitter laser mode $m = 0$ and (b) LD-r1. In (c), we plot the cross-correlation function and in (d) the synchronization error between the two time series.

III. Numerical Results

With the aim of reproducing, qualitatively at least, the experimental observations reported by Lee and Shore [10], we first force the transmitter (multimode) semiconductor laser to have a chaotic output, in a region close the coherence collapse regime [14]. The output intensity is shown in Fig. 2. Although the laser has five longitudinal modes, the power is mainly shared by the three central modes, while the two lateral modes $m = -2$ and $m = +2$ are practically turned off. Different modal dynamics such as in-phase and out-of-phase dynamics can be observed when the pumping intensity is modified [15]. As shown in Fig. 2, for the particular parameters used in the simulations the longitudinal modes show out-of-phase dynamics at fast time scales (see [15] for details).

With the help of an optical isolator, we now unidirectionally inject the light of modes $m = 0$ and $m = +1$ [see Fig. 2(c) and (d)] into two single-mode semiconductor lasers. The coupling strength is set to $k_c = 0.075$ ps^{-1}, which can be considered as strong injection, specially if we compare it with the feedback strength of the transmitter laser $k_t = 0.025$ ps^{-1}. Under these conditions, identical synchronization [16], [17] will never be observed, since the condition $k_c = k_t$ is not fulfilled. Nevertheless, they should be suitable parameters to observe generalized synchronization [18].

One would expect that, since the single-mode semiconductor receiver lasers and the multimode transmitter laser are very different dynamical systems, synchronization would not be achieved. In order to analyze this conjecture, we start studying the simplest case, in which the two receiver lasers have the same internal parameters as the transmitter (except the number of modes, of course). The receiver wavelengths will be slightly different between them, and they will correspond to the wavelengths of two of the modes of the transmitter, in such a way that each individual receiver will only be injected by its corresponding (in wavelength) transmitter mode. In Fig. 3 ,we show the output intensity of the transmitter mode (0) [plot (a)] and the output intensity of the LD-r1 [plot (b)]. At first sight it can be observed that both output powers have certain resemblance. In order to quantify this resemblance, we calculate the cross-correlation function between the two time series, which is given by the expression

$$C(\Delta t) = \frac{\langle (P_t(t) - \langle P_t \rangle)(P_r(t+\Delta t) - \langle P_r \rangle) \rangle}{\sqrt{\left\langle (P_t(t) - \langle P_t \rangle)^2 \right\rangle \left\langle (P_r(t) - \langle P_r \rangle)^2 \right\rangle}} \qquad (7)$$

where P_t and P_r represent the output powers of the transmitter and receiver, respectively, and the brackets indicate time averaging.

The synchronization error will be another good indicator of the similarity between transmitter and receivers, specially if we are interested in message transmission

$$\sigma(\Delta t) = \sqrt{\frac{\left\langle \left(P_t(t) - \frac{P_r(t+\Delta t)}{a} \right)^2 \right\rangle}{\langle P_t^2(t) \rangle}}. \qquad (8)$$

Since the transmitter laser has a feedback strength of $k_c = 0.025$ ps^{-1} while the coupling strength of the receiver is $k_c = 0.075$ ps^{-1}, the latest will have higher power than the former. This difference in power would increase the value of the synchronization error even though both time series display the same dynamics. To reduce this effect, we renormalize the receiver power with the parameter $a = (\langle P_r \rangle / \langle P_t \rangle)$ in the synchronization error function.

Fig. 3(c) shows that the cross-correlation function has a maximum value $C(0) = 0.849$, which reflects the high correlation between the two output powers. The location of the maximum $(\Delta t = 0)$ indicates that we are observing generalized synchronization. On the other hand, the synchronization error has a minimum $\sigma(0) = 0.395$, which seems to indicate that this system will not be suitable for message transmission.

Similar results are obtained with LD-r2 (see Fig. 4), which is also in good correlation with the mode $m = +1$, with a maximum cross correlation $C(0) = 0.855$ and a minimum synchronization error $\sigma(0) = 0.400$.

Some conclusions can be extracted from these numerical simulations. First of all, we have seen that different dynamical systems can show good correlation in their dynamics. Second, since

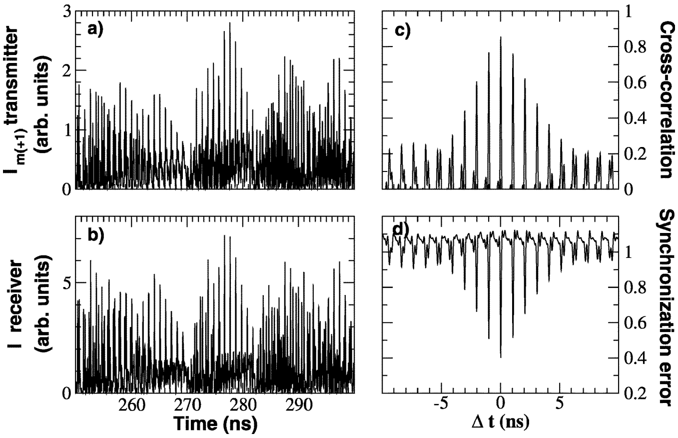

Fig. 4. Output intensity of (a) transmitter laser mode $m = 0$ and (b) LD-r2. In (c), we plot the cross-correlation function and in (d) the synchronization error between the two time series.

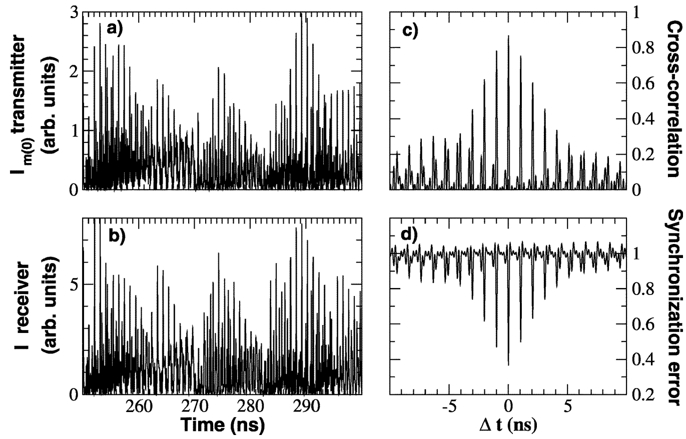

Fig. 5. Output intensity of (a) transmitter laser mode $m = 0$ and (b) LD-r1 with the parameter mismatch indicated in Table II. In (c), we plot the cross-correlation function and in (d) the synchronization error between the two time series.

TABLE II
PARAMETER MISMATCH VALUES

Symbol (Units)	LD-t	LD-r1 (-10%)	LD-r2 ($+10\%$)
α	3.5	3.15	3.85
$\gamma(\text{ ps}^{-1})$	0.238	0.2142	0.2618
$\gamma_e(\times 10^{-4}\text{ ps}^{-1})$	6.211	5.5899	6.8321
$g_c(\times 10^{-9}\text{ ps}^{-1})$	3.2	2.88	3.52
$N_0(\times 10^8)$	1.25	1.125	1.375

the synchronization error is rather high, it seems that the receiver system is reproducing the transmitter dynamics but with different amount of output power. In this case, the output power of the receiver laser will be higher than that of the transmitter, due to the fact that $k_c > k_t$. These observations match with the results obtained by Murakami *et al.* [19], where it was observed a good synchronization for similar conditions but with single-mode semiconductor lasers. Murakami also showed that the synchronization observed is just a consequence of the amplification phenomena of the system, which reflects the injection locking of the receiver.

IV. PARAMETER MISMATCH

At this point we have seen that two completely different dynamical systems, namely a multimode and a single-mode semiconductor laser, can synchronize. Figs. 3 and 4, for example, show a maximum of the cross-correlation function around 0.85 in both cases. This is an acceptable value to consider synchronization between both systems (e.g., see correlations of [20]–[22]). Since injection locking is very tolerant to the parameter mismatch, one could expect to observe a similar level of synchronization when the internal parameters of the receiver lasers are modified. With the aim of confirming this conjecture, the internal parameters of LD-r1 are incremented in 10%, while the parameters of LD-r2 are diminished in 10%. With these modifications, none of the three lasers is identical to each other. The internal parameters of the transmitter and receiver lasers are indicated in Table II.

In Fig. 5, we show the output intensity of mode $m = 0$ and of LD-r1 for the parameter mismatch indicated in Table II. We can observe how the cross correlation has a high value $(C(0) = 0.868)$, even higher than in the case without mismatch $(C(0) = 0.849)$, and at the same time the synchronization error has even decreased. This phenomenon, where the synchronization between two lasers increases when parameters of the receiver laser are slightly decreased, is due to the fact that the injection in the receiver produces an increase of power, and therefore, any change in the internal parameters of the receiver that compensates dynamically for this increase in power will improve the synchronization. The fact that the synchronization of an injected laser can increase for slight parameter mismatches can be observed in [23, Fig. 7]. We obtain similar results in the synchronization of LD-r2 with mode $m = +1$ of the transmitter (see Fig. 6).

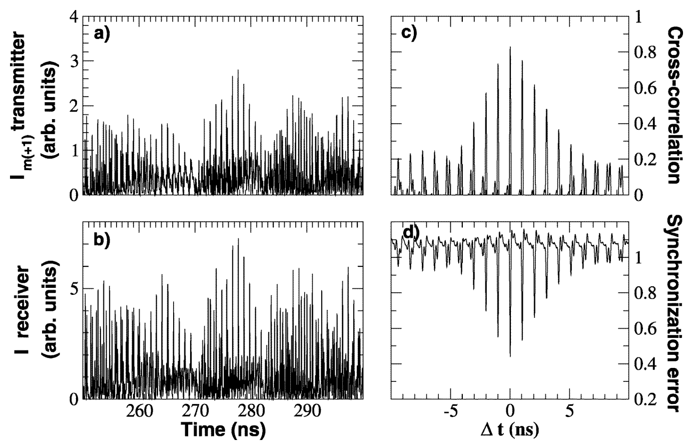

Fig. 6. Output intensity of (a) transmitter laser mode(0) and (b) LD-r2 with the parameter mismatch indicated in Table II. In (c), we plot the cross-correlation function and in (d) the synchronization error between the two time series.

V. MESSAGE TRANSMISSION

We now analyze the potential use of the synchronization scheme described above for encoding information within the chaotic output of the transmitter laser. Since it is possible to synchronize two completely different dynamical systems, namely a multimode and a single-mode semiconductor laser, even with different internal parameters, it would be reasonable to doubt the security of this kind of encryption. If an eavesdropper

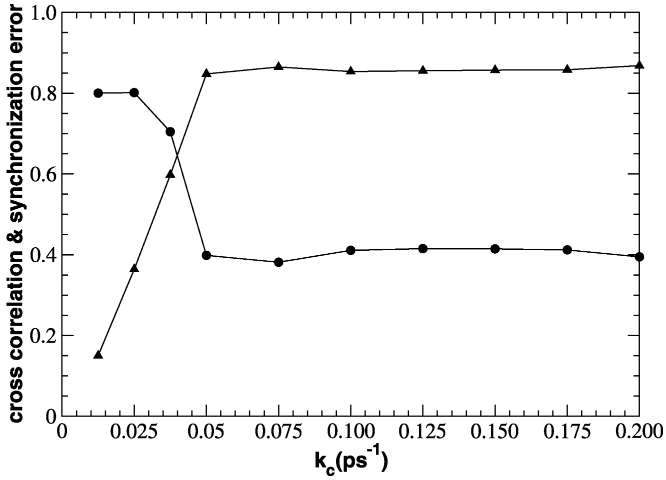

Fig. 7. Cross-correlation function (triangles) and synchronization error (circles) of the output power of the mode $m = 0$ of the transmitter and LD-r1.

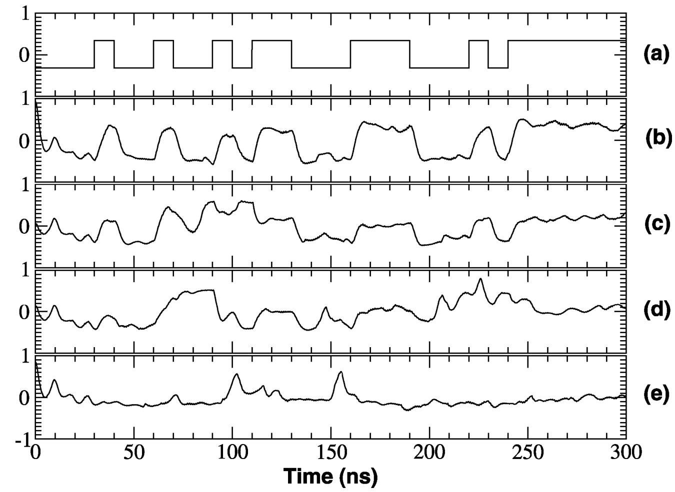

Fig. 8. Input (a) and recovered message for different amplitudes of pump modulation: (b) 16.3%, (c) 10.3%, (d) 7.2%, and (e) 4.5%. All percentages are referred to the threshold pumping current.

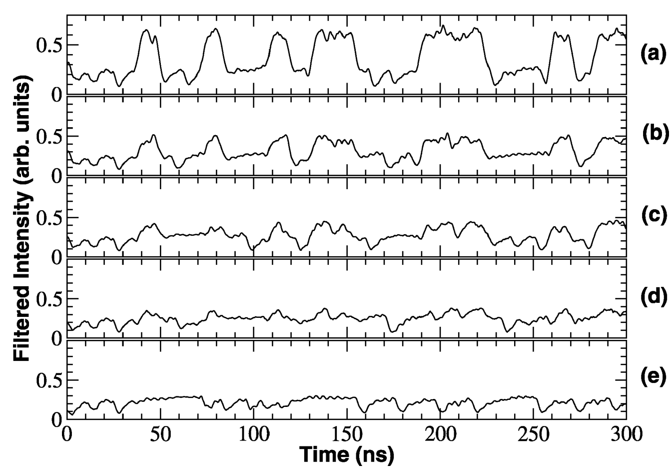

Fig. 9. Output intensity of the transmitter laser filtered by a fourth-order Butterworth filter. The amplitudes of the message (referred to the threshold current) are: (a) 16.3%, (b) 10.3%, (c) 7.2%, (d) 4.5%, and (e) in the absence of message.

with a laser completely different to the transmitter's is able to synchronize their laser output with that of the transmitter, they may able to recover the encrypted message. We are going to check the possibility that an eavesdropper recovers the message with a different laser. We will consider the situation in which a parameter mismatch between transmitter and receivers exist. In principle, a good dynamical correlation between the lasers should be sufficient to recover the message due to the chaos filtering properties of the system. Nevertheless, if nonlinear amplification is the cause of the similarity between the laser outputs, the encoded message will be amplified rather than filtered, which will complicate its recovery in the receiver.

First of all, we are going to evaluate which injection strength is optimal to recover the message. In Fig. 7, we plot the cross-correlation function and the synchronization error for the injection of mode $m = 0$ into LD-r1, with the parameter mismatch of Table II. Since the message is recovered by subtracting both output intensities, the cross-correlation function should be high, but the synchronization error should also be as low as possible. Therefore, we choose a coupling strength of $\kappa_c = 0.050\ \mathrm{ps}^{-1}$, which corresponds with a high cross-correlation and low synchronization error. A further increase of the coupling will not imply a significant increase (decrease) of the cross-correlation (synchronization error) function (see Fig. 7).

We choose as our message an aperiodic bit sequence (at 100 Mb/s), which is injected into the transmitter multimode laser via pump current modulation. We have selected a relatively low frequency of the message due to the fact that the message transmission strongly degrades for high frequencies [23]. On the other hand, the amplitude of the message is another significant parameter from the point of view of communication and encryption. The amplitude of the message is normally kept under 10% of the threshold current of the laser [24], [25]. If the amplitude is too low the message could not be recovered, while for too high amplitudes it would not be hidden by the chaotic carrier. With the aim of finding a suitable message amplitude we have increased its value from 0% to 16% of the threshold current in order to check the performance of the system even for higher message amplitudes. In Fig. 8, we show the recovered message (filtered with a fourth order Butterworth filter) for different message amplitudes. At first sight it seems that the message is accurately recovered for an amplitude of 16.3% [Fig. 8(b)], while for lower amplitudes recovery is difficult to achieve. Nevertheless, the amplitude needed to recover the message is extremely high, a fact that can be observed in Fig. 9, where the output intensity of the transmitter laser is low-pass filtered. For the amplitude of 16.3% [Fig. 9(a)] the message is clearly observed by just filtering the output of the transmitter. Therefore, it would be necessary to go to lower message amplitudes, always lower than 5% [Fig. 9(d) and (e)].

Since we saw in Fig. 8(e) that the message cannot be recovered for a message amplitude of 5% under the described conditions, we now increase the coupling strength κ_c with the aim of improving the recovering. In Fig. 10, we show the recovered message for increasing values of κ_c. One can see that although the coupling is increased, the message is not recovered in any case. In fact, only a nonlinear amplification of the difference between both outputs is observed, denoting that the receiver laser is injection locked. Under the injection locking regime, the receiver laser does not filter the message and its dynamics is a copy of the dynamics of the transmitter. Under these conditions,

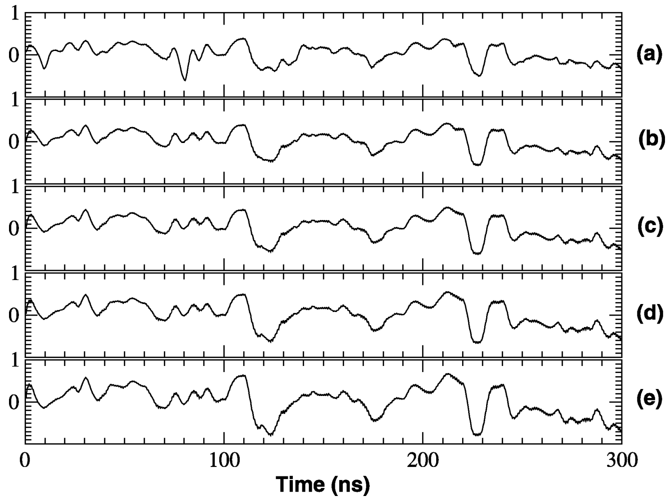

Fig. 10. Recovered message for increasing values of the coupling strength: (a) $\kappa_c = 0.050$ ps^{-1}, (b) $\kappa_c = 0.075$ ps^{-1}, (c) $\kappa_c = 0.100$ ps^{-1}, (d) $\kappa_c = 0.125$ ps^{-1} and (e) $\kappa_c = 0.150$ ps^{-1}. In all cases, the message amplitude is 5% of the threshold current.

although the synchronization is reasonably good, it is not good enough to recover a message encoded via injection-current modulation of the multimode transmitter laser. The loss of the chaos pass filtering properties of the system is due to the high injection rate (compared with the feedback of the transmitter) needed to synchronize the receiver laser with its chaotic input. Under this configuration, the receiver acts as a nonlinear amplifier [19] and follows the dynamics of the transmitter not taking into account if the input is chaotic or periodic.

VI. Conclusion

In conclusion, the results presented above indicate that it is possible to obtain a good correlation between a multimode semiconductor laser and a single-mode one, despite them being two very different dynamical systems. In this kind of correlated dynamics, the match in wavelength between transmitter and receiver laser is a necessary condition, although the synchronization is quite stable to (internal) parameter mismatch, which is a typical feature of injection locking. We have also studied the potential use of this system to transmit an encoded message within the chaotic output of the transmitter laser. The fact that the receiver laser behaves as a nonlinear amplifier, due to the strong injection required to synchronize both systems, leads to inefficient message decoding. This fact shows the security of this kind of hardware encryption versus an eavesdropper with a receiver laser different to the transmitter.

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