NOISE AND COUPLING IN DIODE LASERS

Jordi García-Ojalvo, Javier M. Buldú, Cristina M. González, M. Carme Torrent
Departament de Física i Enginyeria Nuclear
Escola Tècnica Superior d'Enginyeria Industrial de Terrassa
Universitat Politècnica de Catalunya, Spain
jordi.g.ojalvo@upc.edu

Abstract
We review the effects of external fluctuations in semiconductor lasers with feedback, both in isolation and in the presence of coupling from a second laser. Particular attention is paid to the signal detection and processing capabilities of the system. In this sense, a single semiconductor laser with feedback is well known to exhibit stochastic resonance in the response to a single harmonic driving and noise. This behavior has been seen to be substantially enhanced by coupling. Additionally, we have also reported that in the presence of a complex harmonic driving in which the fundamental is missing, the system detects nevertheless the fundamental, in what has been termed ghost resonance. This behavior is also observed in coupled lasers in which the different input harmonics of the complex signals are distributed among the lasers, in a nontrivial example of the signal processing capabilities of the system.

Key words
Diode lasers, optical feedback, excitable lasers, stochastic resonance, ghost resonance, entrainment.

1 Semiconductor lasers as excitable optical devices

Excitable systems are characterized by responding to small perturbations with large pulsed behavior. These pulses have a well-defined shape, irrespective of the input perturbation, provided the latter exceeds a certain threshold. Due to these features, excitable systems are very sensitive to noise, which can trigger spurious pulses (i.e. pulses not produced by a real input signal), when an occasional random fluctuation reaches a large enough amplitude. Additionally, coupling between excitable elements induces in a very straightforward way the propagation of signals, when the output of one excitable element becomes a perturbing driving for a second element. This is, simply put, the way in which information flows within organisms having a nervous system, with neurons being paradigmatic excitable devices. But besides being a communication system, the nervous system (more specifically, the brain) is an extremely efficient signal processing system. An appealing thought is then, can we mimic the outstanding signal processing abilities of the brain and nervous system of higher-order organisms in all-optical photonic communication networks?

In order to answer this question, the first thing that is needed are photonic devices with excitable properties. The quest for such systems has been undertaken in the last decade. Early examples of optical excitability were reported experimentally in CO_2 lasers with saturable absorber (Plaza *et al.*, 1997) and in semiconductor lasers with optical feedback (Giudici *et al.*, 1997), and theoretically in nonlinear optical cavities (Lu *et al.*, 1998), optically injected lasers (Coullet *et al.*, 1998), and in lasers with saturable absorber (Dubbeldam *et al.*, 1999). More recently, excitability was experimentally observed in a solid-state laser with saturable absorber (Larotonda *et al.*, 2002). Further theoretical developments include the prediction of multipulse excitability in a laser with optical injection (Wieczorek *et al.*, 2002) and the description of a mechanism of excitability via localized structures in a nonlinear optical cavity (Gomila *et al.*, 2004). Spatiotemporal excitable structures have been reported experimentally in optically injected broad-area semiconductor microcavities (Marino and Balle, 2005).

Networks of lasers have been proposed as basic architectures in the context of optical neurcomputing (Hoppensteadt and Izhikevich, 2000) and neural networks (Mos *et al.*, 2000). However, up to our knowledge no practical implementations of these technologies have been realized so far. Here we present both theoretical and experimental results showing the potential of semiconductor lasers for performing basic tasks in signal detection and processing, discussing in particular the role of noise and coupling. The experimental system considered is a semiconductor laser with optical feedback, whose excitable-like properties were first described experimentally by (Giudici *et al.*, 1997), as mentioned above, and theoretically by (Mulet and Mirasso, 1999).

2 Signal detection and processing by excitable systems

We will concentrate in two basic mechanisms of signal detection and processing that are known to be exhibited by excitable systems, namely stochastic resonance and ghost stochastic resonance.

2.1 Stochastic resonance

Stochastic resonance consists on the optimal detection of a weak signal with the aid of an appropriate level of noise. It is well known to occur in simple bistable devices (Wiesenfeld and Moss, 1995; Gammaitoni *et al.*, 1998; Neiman *et al.*, 2001), and also in excitable systems (Lindner *et al.*, 2004). Stochastic resonance has been proposed as a mechanism of weak signal detection in living beings, where random fluctuations are expected to play an important role given the large amount of noise present in neural tissue. A large number of experimental studies, both behavioral and physiological, have shown the relevance of stochastic resonance for signal detection in living systems (see (Lindner *et al.*, 2004) for a review).

In the particular case of semiconductor lasers with optical feedback, stochastic resonance was reported by (Marino *et al.*, 2002). We review this phenomenon below from a theoretical perspective (Sect. 3), and later we show experimentally that the response to a weak harmonic signal can be greatly enhanced by coupling (Sect. 4).

2.2 Ghost stochastic resonance

The simplest case of signal processing involves subjecting the excitable element to two inputs and extracting a single output. This is routinely performed, for instance, by the hearing sensory system of higher-order organisms when perceiving the pitch of a complex sound (von Helmholtz, 1895). Experiments have been conducted that subject human beings to complex harmonic sounds (sets of harmonics whose fundamental is absent), and the results show that the individuals perceive the fundamental tone even though it is not among the input signals (Schouten *et al.*, 1962). This phenomenon is known as the “missing fundamental illusion”.

Recently a mechanism for this illusion has been proposed in terms of stochastic resonance. The mechanism relies on the linear superposition of the input signals and the noise-aided detection of the processed signal (Chialvo *et al.*, 2002), and has been termed “ghost stochastic resonance”, since a frequency that is not really there (*i.e.* a ghost) is detected.

As described in Sect. 5, we have shown experimentally that the phenomenon of the ghost resonance exists in excitable semiconductor lasers. In a further non-trivial development, described in Sect. 6, it has been observed that a similar behavior occurs when the inputs are distributed in different excitable elements, so that signal processing is mediated by coupling.

3 Signal detection is enhanced by noise

We first consider the setup shown schematically in Fig. 1, in which a semiconductor laser is subject to optical feedback from an external mirror, and its injection current is modulated by a harmonic signal plus noise. Due to the feedback (assumed moderate), when operating close to threshold the laser undergoes the so-called *low-frequency fluctuations*. These fluctuations take the form of power dropouts (*i.e.* inverted pulses) that qualitatively exhibit the main characteristics of an excitable system, namely, they can arise as strong pulsed responses to small suprathreshold perturbations, the pulse shape being basically independent of the input perturbation (Giudici *et al.*, 1997; Mulet and Mirasso, 1999).

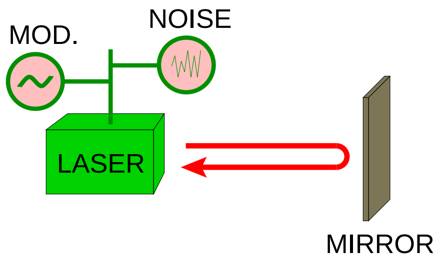

Figure 1. Semiconductor laser under optical feedback and harmonic+random modulation of the pump current.

This behavior can be described in a simplified way by the single-mode, single-reflection Lang-Kobayashi model:

$$\frac{dE}{dt} = \frac{1+i\alpha}{2}\left(G(E,N)-\gamma\right)E(t) + \kappa e^{-i\omega\tau_f}E(t-\tau_f) + \sqrt{2\beta N}\zeta(t) \quad (1)$$

$$\frac{dN}{dt} = \gamma_e[C(t)N_{\rm th} - N(t)] - G(E,N)\,|E(t)|^2\,,$$

where γ and γ_e are the inverse lifetimes of photons and carriers, respectively, α is the linewidth enhancement factor, and ω is the free running lasing frequency. The pumping term $C(t)$ has the form $C(t) = C_0[1+\xi(t)+A\sin(\Omega t)]$, where C_0 is the bias pumping rate (directly related to the DC driving current; $C=1$ is the solitary-laser threshold). Pumping is affected by a random term (represented by $\xi(t)$) and a harmonic driving of amplitude A and frequency Ω. The last term in the electric-field equation represents spontaneous emission fluctuations, with $\zeta(t)$ given by a Gaussian white noise of zero mean and unity intensity, and β measuring the internal noise strength. The material-gain function $G(E,N)$ is given by

$$G(E,N) = \frac{g(N(t)-N_0)}{1+s|E(t)|^2}\,, \quad (2)$$

where g is the differential gain coefficient, N_0 is the carrier number in transparency and s the saturation coefficient. The threshold carrier number is $N_{\rm th} = \gamma/g + N_0$. Finally the optical feedback term is described by two parameters: the feedback strength κ and the external round–trip time τ_f.

The low-frequency power dropouts occur at irregular times. A large effort has been devoted to control this dynamics, for instance to render the dropouts periodic in time via an external harmonic modulation of the pump current. But the modulation amplitude needed to entrain the dropouts has to be quite large, distorting significantly the dynamics (Sukow and Gauthier, 2000) (see also the top plot in Fig. 5 below). In the case of small-amplitude periodic driving, an intermediate amount of noise can lead to an optimal entrainment of the low frequency fluctuations to the external modulation. This is shown in Fig. 2, which plots the temporal behavior of the intensity and phase difference in one external roundtrip (related to the instantaneous frequency) for three levels of noise. As shown in the figure, the power dropouts coincide with sudden jumps in the phase difference, and these jumps are clearly better entrained to the periodic driving (with a 50 ns period in this case) for an intermediate noise level (middle plot).

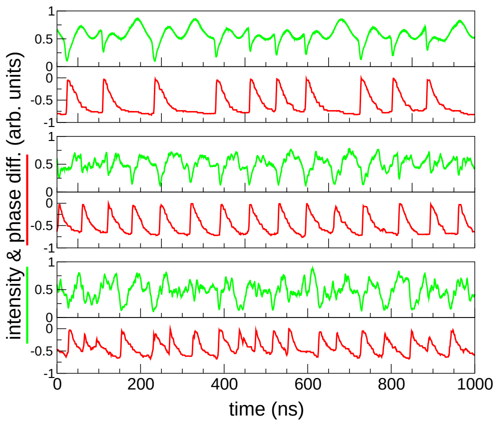

Figure 2. Stochastic entrainment of low-frequency fluctuation pulses. Noise intensity increases from top to bottom. Numerical results.

An important consideration here is that the electronic noise used in this type of experiments will have a bandwidth necessarily smaller than the typical frequencies of the laser dynamics (which is higher than tens of gigahertz). Therefore the correlation time of the noise is not negligible in these studies. The result shown in Fig. 2 corresponds in fact to a correlation time on the order of tens of picoseconds, which is optimal for signal detection (Buldu *et al.*, 2002*a*).

4 Signal detection is enhanced by coupling

As mentioned above, a large level of modulation is required in order to entrain the power dropouts to an external periodic driving (signal to be detected). This produces a large distortion of the laser dynamics upon entrainment. Besides noise, a second mechanism known to enhance the response of a nonlinear system to external driving is coupling (Lindner *et al.*, 1995).

Consider the experimental setup shown in Fig. 3. Two lasers are facing each other and mutually inject their output radiation into one another, in a straightforward scheme of bidirectional coupling. It is known that in such a configuration, the lasers can enter a low-frequency fluctuation regime analogous to the one observed in a single laser with optical feedback (Heil *et al.*, 2001).

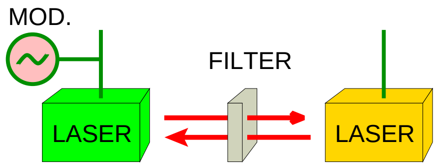

Figure 3. Two bidirectionally coupled semiconductor lasers. A weak harmonic modulation of the pump current is applied to one of the lasers.

We now ask ourselves what is the response of this system to a harmonic driving of the pump current of one of the lasers. The result is shown in Fig. 4 for increasing levels of coupling. It is clear that the pulses become more pronounced as coupling increases, an indication that it is coupling that leads to the instability in the first place. Furthermore, and also understandably, as coupling increases, the synchronization between the dynamics of the two lasers improves. Finally, for large

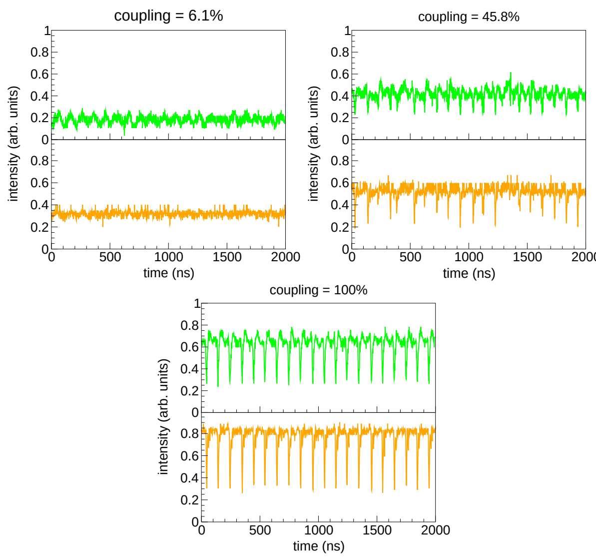

Figure 4. Response of two mutually injected lasers to harmonic modulation of the pump current of one of them, for increasing coupling. Coupling increases from left to right and from top to bottom. Experimental results. Coupling percentages refer to the maximum coupling level available experimentally.

enough coupling we see a very good periodic entrainment of the power dropouts (the driving period is here 100 ns).

A comparison between the entrainment obtained purely by direct modulation (Sukow and Gauthier, 2000) and the one produced by modulation and coupling (Buldu *et al.*, 2002*b*) is displayed in Fig. 5. We note that a much smaller level of modulation is needed for the coupled laser case (bottom plot, see plot title) than for the direct modulation case (top plot, see plot title). This leads, of course, to a much smaller perturbation of the overall dynamics in the coupled case.

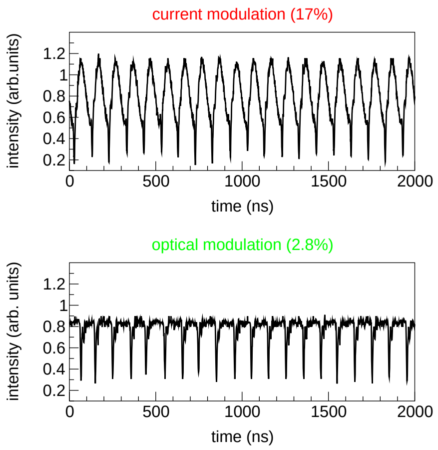

Figure 5. Comparison between the periodic entrainment of low-frequency fluctuations in a single laser with optical feedback (top) and two coupled lasers (bottom). Experimental results.

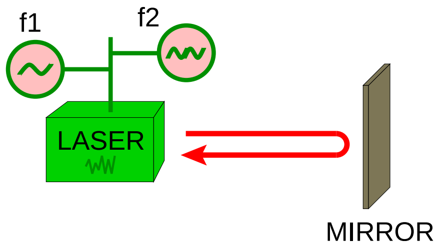

Figure 6. Semiconductor laser under optical feedback and a complex harmonic modulation of the pump current (see text).

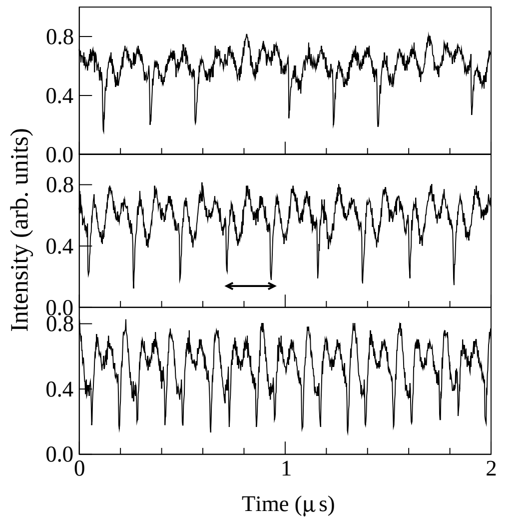

Figure 7. Ghost stochastic resonance of low-frequency fluctuation pulses. The horizontal arrow in the middle plot indicates the period of the missing fundamental. Driving amplitude increases from top to bottom.

5 Signal processing is enhanced by noise

As described in Sect. 2.2, we now consider the simplest case of signal processing, in which the laser is subject to a complex harmonic signal composed of two frequencies of the form:

$$f_n = (k + n - 1)f_0 + \Delta f, \qquad n = 1, 2, \quad (3)$$

where $k > 1$ is an integer and Δf is a frequency detuning. The scheme is depicted in Fig. 6. Again the excitable element is a semiconductor laser with optical feedback from an external mirror, operating in the low-frequency fluctuation regime.

We don't include external noise in the experimental studies described below, but rather use the "internal" noise arising from the complex dynamics of the low-frequency fluctuation regime. The experimental response of the laser to an increasing amplitude of the modulation at the two frequencies is shown in Fig. 7, for a harmonic set of two signals with $k = 2$ and $\Delta f = 0$ in Eq. (3).

The horizontal arrow in the middle plot of Fig. 7 corresponds to the frequency of the fundamental (4.5 MHz), which is not present in the input signal set (9 MHz and 13.5 MHz). The laser is not merely processing the frequency difference between the two harmonics, because in the case of an inharmonic input [$\Delta f \neq 0$ in Eq. (3), for which the frequency difference is still $f_0 = 4.5$ MHz, but where f_1 and f_2 are no longer harmonics of f_0] the perceived frequency varies as (Buldu *et al.*, 2003):

$$f_r = f_0 + \frac{\Delta f}{k + 1/2}. \quad (4)$$

i.e. linearly with Δf.

6 Signal processing is enhanced by coupling

Finally, we examine the potentially beneficial role of coupling in the signal processing scheme described above. As seen in Sect. 4, coupling greatly enhances the response of the laser to a pure harmonic modulation, and hence we can expect a similar behavior in the case of a *complex* harmonic modulation. Furthermore, in this case we can examine the situation in which the drivings corresponding to the different harmonics of the complex signal are applied to *different* lasers. In other words, we consider the signals to be *distributed* in the nodes of the processing network (in this case composed of only two elements, subject to one signal each). The experimental setup is schematically represented in Fig. 8.

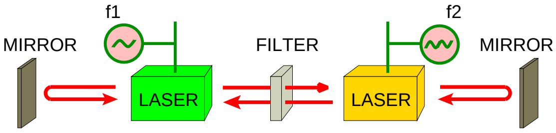

Figure 8. Two lasers with optical feedback coupled bidirectionally.

The two processing elements are again semiconductor lasers with optical feedback (as opposed to the situation considered in Sect. 4, where no mirrors where considered and the instability was due only to the coupling between the lasers; in the present case the individual lasers are potentially excitable even in the absence of coupling). We stress again that each one of the harmonics is applied to a different laser; if the missing fundamental is perceived in this case, it will be due to the interplay between the directly modulated electrical driving and the indirect driving coming from the other laser (which has nonlinearly filtered its own current modulation). Hence the processing in this case, if it happens, is extremely non-trivial.

Figure 9 shows the response of the two lasers under these conditions, both in the presence of coupling (top plot) and in its absence (middle and bottom plots). It

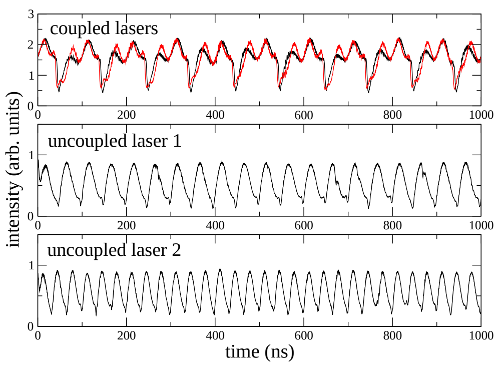

Figure 9. Response of the coupled lasers to the complex harmonic signal (top) compared with the responses of the two lasers in the absence of coupling (middle and bottom).

is clearly observed that in the presence of coupling the period of the missing fundamental (100 ns) is extracted, while when the lasers are isolated from each other only the corresponding periods of the individual modulations (50 ns and 33 ns, respectively) are detected. A similar result is obtained in the absence of mirrors (Buldu *et al.*, 2005). In other words, the lasers are able to process two signals coming from very different paths (one from direct electrical modulation and the other from optical injection from the other modulated laser) and respond in a self-organized (and synchronized) way at a frequency not present in the distributed inputs.

7 Conclusion

We have shown, via simple examples of signal detection and processing, that excitable optical elements are promising unit devices in all-optical data processing networks. In particular, the use of semiconductor lasers is attractive due to their low cost, wide availability and ease of integration. New directions of research should include further characterization of the influence of the coupling architecture of the laser network, and considering more complex detection and processing tasks.

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